Section 2.3 Continuity

The graph of a function can be roughly sketched given only specific limits and function values.

# Topic 1: Continuity

A function *f* is **continuous at a point**  if the following three properties are true.

1.  exists
2.  exists
3. 

A function *f* is **continuous on the open interval ** if it is continuous at each point on the interval. If a function is not continuous, it is **discontinuous**.

Rather than list the points where a function is discontinuous, sometimes it is useful to state the intervals on which the functions is continuous. Using the set operation union, denoted , we can express the set of points where the function is continuous.

# Topic 2: Continuity Properties

**Theorem: Continuity Properties of Some Specific Functions.**

* A constant function , where *k* is a constant, is continuous for all values of *x*.
* For *n*  a positive integer,  is continuous for all values of *x*.
* A polynomial function is continuous for all values of *x*.
* A rational function is continuous for all values of *x* except those values that make the denominator 0.
* For *n* an odd positive integer greater than 1,  is continuous wherever *f* is continuous.
* For *n* an even positive integer,  is continuous wherever *f* is continuous and nonnegative.

# Topic 3: Solving Inequalities Using Continuity Properties

A tool for analyzing graphs of functions, or for solving inequalities, is the **sign chart**. We find where the function is zero or undefined, then partition the number line into intervals at these points. We then test each interval to determine if the function is positive (above the *x*-axis) or negative (below the *x*-axis) in those intervals.

**Constructing a Sign Chart** for a function

1. Find all partition numbers of :
2. Find all numbers *x* such that *f* is discontinuous at *x*. (Rational functions are discontinuous at values of *x* that make the denominator 0).
3. Find all numbers *x* such that . $ $(For rational functions, this occurs where the numerator is 0 and the denominator is not 0).
4. Plot the numbers found in step 1 on a real number line, dividing the number line into intervals.
5. Select a test number in each open interval determined in step 2 and evaluate  at each test number to determine whether  is positive (+) or negative (–) in each interval.
6. Construct a sign chart, using the real number line in step 2. This will show the sign of *f* on each open interval.