# MCT'03

# Louisiana Conference on Mathematical Control Theory

**Louisiana State University**
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**Peter R. Wolenski, Conference Chair**

**Book of Abstracts**

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# Plenary Talks

**Francis Clarke** (clarke@desargues.univ − lyon1.fr), Institut Girard Desargues, Universit Claude-Bernard (Lyon I), France, *The Euler Equation in the Calculus of Variations and Optimal Control: A Survey*

The various forms and uses of the Euler equation in dynamic optimization are reviewed, including some new results on the issue of necessary conditions.

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**R. Tyrrell Rockafellar** (rtr@math.washington.edu), Department of Mathematics, University of Washington, *Feedback and Cost-to-Go in Control Problems of Convex Type*

A connection between optimal feedback and the cost-to-go function in problems of optimal control has long been recognized, but efforts at putting this into practice have largely been limited to the classical case of linear-quadratic control with no state or control constraints. Recent developments reveal that, for a much broader class of control models exhibiting convexity in states and controls, unusually favorable properties are likewise present which well might be put to use. Such properties come out of the analysis of the subgradient mapping for the cost-to-go function and various Hamiltonian characterizations of the way it evolves. In principle, they support nonlinear global feedback rules which allow not only for constraints but also for penalty expressions that may be just piecewise smooth.

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**Héctor J. Sussmann** (sussmann@hilbert.rutgers.edu), Department of Mathematics, Rutgers University, *Finite-Dimensional Generalized Differentiation Theories that Satisfy the Chain Rule and Have an Open Mapping Property*

The classical differential of maps of class $C^1$ is a functor from the category $PMC^1$ of pointed manifolds of class $C^1$ (i.e., pairs $(M, p)$ consisting of a manifold $M$ of class $C^1$ and a distinguished point $p$ of $M$) and distinguished-point-preserving maps of class $C^1$ to the category $FDRLS$ of finite-dimensional real linear spaces and linear maps. Furthermore, this functor has an open mapping property: if the differential of a map $F$ at a point $p$ is surjective, then the map itself sends neighborhoods of $p$ to neighborhoods of $F(p)$. We study extensions of this functor to categories with the same objects as $PMC^1$ but larger classes of morphisms (including Lipschitz maps, many continuous maps that are not Lipschitz, and many set-valued maps), taking values in a larger category (whose morphisms are nonempty compact sets of linear maps), and still having the open mapping property in a suitable sense. These extensions are really "multivalued functors" (as might perhaps have been expected for a theory that belongs to the general field of set-valued analysis), and the functoriality property is the Chain Rule. A number of such extensions had been constructed in previous work by several authors, including the speaker, but they were not mutually comparable. We propose an extension, called "path-integral generalized differentials", that contains all the others and still obeys the chain rule and has an open mapping property. We also discuss the harder problem of working with a category with a larger class of objects, e.g. pairs $(C, p)$ where $C$ is a subset of a manifold $M$ of class $C^1$ which is, near $p$, $C^1$-diffeomorphic to a closed convex cone, and show how the theory of path-integral generalized differentials also work in this case, at least if one limits oneself to sets $C$ that are $C^1$-diffeomorphic to polyhedral cones.

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**Andrew R. Teel** (`teel@ece.ucsb.edu`), Department of Electrical and Computer Engineering, University of California-Santa Barbara, Santa Barbara, CA 93106-9560, *Discrete Time Receding Horizon Optimal Control: Is the Stability Robust?*

Receding horizon or model predictive control is an optimization-based paradigm for stabilizing nonlinear systems. It is especially well-suited for systems that are subject to input and/or state constraints. Model predictive control is commonly used in industry, especially the chemical processing industry where the driving time scales are sufficiently slow. In engineering circles, it is most frequently presented in discrete time, which will be the focus of this talk. Frequently the feedback algorithm derived from model predictive control is discontinuous. While this doesn't necessarily cause problems, some discontinuous feedback algorithms have absolutely no robustness. That is to say, arbitrarily small disturbances can keep the trajectories a fixed distance from the equilibrium point that presumably was stabilized. We will show that, for certain systems with certain state constraints, model predictive control has this unfortunate feature. After this, we will show how some problems that involve state constraints can be recast so that robust stability is guaranteed. Along the way, we will show how the typical requirements on the components of the underlying optimization problem in model predictive control can be relaxed. If time permits, we will conclude with a discussion relating robustness to the existence of a continuous (in fact, smooth) Lyapunov function.

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**Richard Vinter** (`r.vinter@ic.ac.uk`), Department of Electrical and Electronic Engineering, Imperial College, London, UK, *Differential Games and Controller Design: A Case Study in Process Control*

This presentation concerns a class of dynamic optimization problems, which we term exit problems, where the object is to contain the state in an allowed region for as long as possible, in the presence of disturbances. The theory relating to such problems has widespread applications. Exit from the allowable region represents, for example, the saturation of a communication channel, loss of 'lock' in a radio communications link or, in process control, an undesirable change of phase, overflow, etc.. We review various formulations of exit problems that have been proposed, notably stochastic and game theoretic formulations, and inter-relate them. Special attention is given to a design problem in process control. It is argued that approaching this design problem as an example of an exit problem takes account of disturbances in a very natural way and offers improvements over other approaches. An explicit solution to this problem is given. This example is of interest, partly because it provides a new design tool in process control, but also because of the insights it gives in the kinds of pathologies that the general theory must address.

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# Other Talks

**Fabio Ancona** (ancona@ciram.unibo.it), Department of Mathematics and C.I.R.A.M., Università di Bologna, Piazza Porta S. Donato 5, Bologna 40127, Italy, *Stabilization by Patchy Feedbacks and Robustness Properties*

This talk is concerned with the problem of constructing discontinuous stabilizing feedbacks for nonlinear control system, which enjoy robusteness properties with respect to external and internal perturbations (cf. [3,4]). We first consider "patchy" vector fields, a class of discontinuous, piecewise smooth vector fields introduced in [1], and we prove the stability of the corresponding solution set with respect to impulsive perturbations. A linear estimate of the effect produced by such perturbations is also established for a generic class of patchy vector fields in the plane, that admit discontinuities across polygonal lines. Next, we apply these results to derive robusteness properties with respect to both (internal) measurement errors and persistent external disturbances for "patchy feedbacks": a class of feedback laws that generate patchy vector fields. This talk is based on joint work [1,2] with Alberto Bressan.

[1] Ancona, F., and A. Bressan, "Patchy vector fields and asymptotic stabilization," *ESAIM: Control, Optimiz. Calc. Var.,* **4**(1999), pp. 445-471.

[2] Ancona, F., and A. Bressan, "Flow Stability of Patchy Vector Fields and Robust Feedback Stabilization," *SIAM J. Control Optim.*, **41**(2003), pp.1455-1476.

[3] Clarke, F., Yu.S. Ledyaev, L. Rifford, and R. Stern, "Feedback stabilization and Lyapunov functions," *SIAM J. Control Optim.*, **39**(2000), pp. 25-48.

[4] Sontag, E., "Stability and stabilization: discontinuities and the effect of disturbances," in *Proc. NATO Advanced Study Institute - Nonlinear Analysis, Differential Equations, and Control*, (Montreal, Jul/Aug 1998), F.H. Clarke and R.J. Stern eds., Kluwer, 1999, pp. 551-598.

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**Zvi Artstein** (zvika@wisdom.weizmann.ac.il), Department of Mathematics, Weizmann Institute of Science, Rehovot, Israël, *Switches and Impulses Induced by Singular Perturbations*

The limit as the small parameter tends to zero of a singularly perturbed system may produce switching of modes and impulses of the slow dynamics. These may be of benefit in an optimal control framework. Conditions for, and examples of, the phenomena will be displayed.

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**Boris P. Belinskiy*** (`bbelinsk@cecasun.utc.edu`), University of Tennessee-Chattanooga, and **Sergei A. Avdonin** (`ffsaa@uaf.edu`), University of Alaska-Fairbanks, *Some New Developments in Exact Control Theory and the Method of Moments*

We study the exact controllability for a flexible elastic string fixed at the end points under an axial stretching tension that slowly varies in time. We say that the string is controllable if, by suitable manipulation of the transverse load, the string goes to the rest. We are looking for an exterior transverse load that drives the state solution to the rest. To prove our results we apply the method of moments that has been widely used in control theory for distributed parameter systems (cf. the classical papers of H.O. Fattorini and D.L. Russell). The problem of exact controllability is reduced to a moment problem for the control. The proof of controllability is based on an auxiliary basis property result that is of independent interest.

The results of this paper may be considered as a generalization of the classical results on controllability for one-dimensional wave equation. The main difference between our problem of control and the classical results is that in our case, the coefficient of the wave equation (tension in our model) is a function of time. As a result, the functions that substitute non-harmonic exponential functions may not be found explicitly. This fact complicates the analysis of controllability. We use some results on the bases of non-orthogonal functions due to M.G. Krein. To the best of our knowledge, our work is the first attempt to apply the method of moments to equations with time dependent coefficients. We outline some possible applications (such as control of oscillations of a system of connected thin elastic cylinders imitating blood vessels, control that would stabilize unnecessary oscillations of a propeller of a helicopter, etc.).

The research of the second author was supported by a University of Tennessee at Chattanooga Center for Excellence in Computer Applications Scholarship.

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**Gaemus Collins** (`gcollins@ucsd.edu`), Department of Mathematics, University of California, San Diego, CA 92093-0112, *Min-Plus Eigenvector Methods for Nonlinear $H_\infty$ Problems with Active Control*

In [M] McEneaney considers the $H_\infty$ problem for a nonlinear system. The dynamic programming equation (DPE) is a fully nonlinear, first order, steady state partial differential equation (PDE), posessing a term which is quadratic in the gradient. The solutions are typically nonsmooth, and further, there is non-uniqueness among the class of viscosity solutions. In the case where one tests a fixed-feedback control to see if it yields an $H_\infty$ controller, the PDE is a Hamilton-Jacobi-Bellman equation. In the case where the "optimal" feedback control is being determined as well, the problem takes the form of a differential game, and the PDE is, in general, an Isaacs equation. The computation of the solution of a nonlinear, steady-state, first-order PDE is typically quite difficult. In [M] McEneaney began the development of an entirely new class of methods for obtaining the "correct" solution of such PDEs. The focus was on the fixed-feedback case. The methods were based on the linearity of the associated semigroup over the max-plus algebra. In particular, the solution of the PDE was reduced to the solution of a max-plus eigenvector problem for the known unique eigenvalue 0 (the max-plus multiplicative identity). It was demonstrated that the eigenvector is unique and that the power method converges to it.

McEneaney briefly considers the problem with active control, that is, where the feedback is unknown. It is conjectured that similar results to the fixed-feedback control case should hold. In this paper the corresponding value function for the active control problem is shown to satisfy a similar PDE as in the fixed-feedback control case. We assume that the controller can dominate the disturbance input. We prove the semigroup associated with this new PDE is linear over the min-plus algebra, the solution is semi-concave, and the associated min-plus eigenvalue problem not only gives the solution to the PDE, but can also be solved by the power method.

[M] W. McEneaney, "Max-plus eigenvector representations for solution of nonlinear $H_\infty$ problems: basic concepts," *IEEE Transactions on Automatic Control*, submitted.

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**Giovanni Colombo*** (colombo@math.unipd.it), Dipartimento di Matematica Pura ed Applicata, Università di Padova, Italy, and **Peter Wolenski** (wolenski@math.lsu.edu), Department of Mathematics, Louisiana State University, USA, *On a Class of Generalized Distances in Hilbert Spaces*

Some properties which are known for distance and metric projections in Hilbert spaces are extended to a class of minimum time problems with constant dynamics but not necessarily convex target. Fréchet and proximal subgradients of a minimum time function are explicitly computed, under various assumptions on the dynamics and on the target. Special emphasis is placed on the regularity properties of a generalized metric projection.

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**Luca Consolini*** (luca.consolini@polirone.mn.it) and **Mario Tosques** (mario.tosques @unipr.it), Università degli Studi di Parma, Italy, *Locally Controlled Invariance of a Manifold for Nonlinear Systems*

This paper presents a sufficient condition for a manifold $\Gamma$ to be locally controlled invariant at $x_0 \in \Gamma$ which reduces, in the cases of linear and nonlinear affine systems, to well known results in the literature. Essentially, the result says that a manifold $\Gamma \subset \mathbf{R}^n$ is locally controlled at $x_0 \in \Gamma$ if, first of all, we can find a control $u_0 \in \mathbf{R}^m$ such that $F(x_0, u_0) \in T_{x_0}\Gamma$ (this condition being evidently necessary), and second, $F(x, u)$ continues to stay in something larger than $T_x\Gamma$ (namely $T_x\Gamma + \partial_u F(x, u)(\mathbf{R}^m)$) in a neighborhood of $(x_0, u_0)$ in $\Gamma \times \mathbf{R}^m$.

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**M. d. R. de Pinho*** and **M. M. A. Ferreira** ({mrpinho, mmf}@fe.up.pt), ISR and Faculdade de Engenharia da Universidade do Porto, FEUP-DEEC, Rua Dr. Roberto Frias, 4200-465 Porto, Portugal, and **F. A. C. C. Fontes** (ffontes@math.uminho.pt), Universidade do Minho, 4800-058 Guimarães, Portugal, *Euler-Lagrange Inclusions for Optimal Control Problems*

Necessary conditions of optimality in the form of an Euler Lagrange Inclusion (ELI) are derived for optimal control problems with state constraints. The conditions presented here generalize earlier optimality conditions to problems that may be nonconvex. The removal of the convexity assumption is of particular importance when deriving ELI type conditions. We illustrate this feature by deriving necessary conditions for problems that in addition to state constraints incorporate mixed state-control constraints.

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**M.S. de Queiroz*** (`dequeiroz@me.lsu.edu`), Department of Mechanical Engineering, LSU; and **B. Xian** and **D.M. Dawson** ({`xbin`, `ddawson`}`@ces.clemson.edu`), Department of Electrical and Computer Engineering, Clemson University, *A Continuous Control Mechanism for Asymptotic Tracking of Uncertain Multi-Input Nonlinear Systems*

In this talk, we present a novel continuous control mechanism that compensates for uncertainty in a class of multi-input nonlinear systems while ensuring semi-global asymptotic tracking. The control mechanism is based on limited assumptions on the structure of the system nonlinearities, and has the interesting feature of learning the unknown system dynamics. Simulation results illustrate the performance of the proposed control in comparison to a variable structure controller.

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**Tzanko Donchev** (`tdd51us@yahoo.com`), Department of Mathematics, University of Architecture & Civil Engineering, 1046 Sofia, Bulgaria, *Relaxed One Sided Lipschitz Multifunctions and Applications*

We present an overview of the so-called Relaxed One Sided Lipschitz (ROSL) multifunctions and their applications to differential inclusions

$$\dot{x}(t) \in F(t, x(t)),\ x(0) = x_0,\ t \in I := [0, 1]. \tag{1}$$

Given a Hilbert space $E$, a bounded valued multifunction $F$ from $I \times E$ into $E$ is said to be **Relaxed One Sided Lipschitz (ROSL)** with a constant $L$ provided

$$\sigma(x - y, F(t, x)) - \sigma(x - y, F(t, y)) \leq\ L|x - y|^2 \tag{2}$$

for a.e. $t \in I$ and each $x, y \in E$, where $\sigma(\cdot, B)$ is the support function of the set $B$. If $E$ is a Banach space with uniformly convex dual $E^*$, then the inequality (2) becomes

$$\sigma(J(x - y), F(t, x)) - \sigma(J(x - y), F(t, y)) \leq\ L|x - y|^2,$$

where $J(\cdot)$ is the normalized duality map. When $E$ is an arbitrary Banach space, however, one must use the following definition:

*For every $x, y \in E$ and a.e. $t \in I$:*
*If $f_x \in F(t, x)$, then for every $\varepsilon > 0$ there exists $f_y \in F(t, y)$ such that*

$$[x - y, f_x - f_y]_+ < L|x - y| + \varepsilon,$$

*where $[x, y]_+ = \lim\limits_{h \to 0^+} h^{-1}\{|x + hy| - |x|\}$.*

It is easy to see that the ROSL condition is essentially weaker than Lipschitz continuity and relaxes essentially the "classical" OSL condition:

$$[x - y, f_x - f_y]_+ \leq L|x - y|,\ \forall f_x \in F(t, x),\ f_y \in F(t, y).$$

Moreover, if $F$ is Lipschitz, then it is possible that the ROSL constant can be less than the Lipschitz one. The following theorem is a standard result for the differential inclusion (1):

**Theorem:** *Let* $\operatorname{co} F(\cdot,\cdot)$ *be almost continuous, with nonempty compact values, and bounded on bounded sets. If* $F(t,\cdot)$ *is ROSL with closed values, then the solution set of*

$$\dot{x}(t) \in \operatorname{co} F(t, x(t)),\ x(0) = x_0 \tag{3}$$

*is a nonempty* $R_\delta$ *set. Furthermore, the solution set of (1) is nonempty, connected and dense in the solution set of (3).*

We consider several other applications of the ROSL condition and present illustrative examples.

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**A. L. Dontchev** (ald@ams.org), Mathematical Reviews, Ann Arbor, MI 48107-8604, *An Inverse Function Theorem for Metrically Regular Mappings*

We prove that if a mapping $F : X \rightrightarrows Y$, where $X$ and $Y$ are Banach spaces, is metrically regular at $\bar{x}$ for $\bar{y}$ and its inverse $F^{-1}$ is convex and locally closed-valued around $(\bar{x}, \bar{y})$, then for any function $G : X \to Y$ with $\operatorname{lip} G(\bar{x}) \cdot \operatorname{reg} F(\bar{x}\,|\,\bar{y}) < 1$, the mapping $(F + G)^{-1}$ has a continuous local selection $x(\cdot)$ around $(\bar{x}, \bar{y} + G(\bar{x}))$ which is also calm.

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**Tzanko Donchev** (tdd51us@yahoo.com), University of Architecture & Civil Engineering, 1046 Sofia, Bulgaria, **Elza M. Farkhi*** (elza@post.tau.ac.il), School of Mathematical Sciences, Tel Aviv University, Israel, and **Peter Wolenski** (wolenski@math.lsu.edu), Department of Mathematics, Louisiana State University, *Characterizations of Reachable Sets for a Class of Differential Inclusions*

This paper studies a differential inclusion under the relaxed one-sided Lipschitz condition, and provides a characterization of its multivalued reachable set mapping. Previous characterizations of reachable set mappings, such as in the well-known funnel equation, have been local and require a “two-sided” Lipschitz assumption. Under the weaker one-sided assumption, the characterization in this paper has a global character, and in particular, identifies a class in which unform limits of Euler polygonal arcs coincide with the absolutely continuous solutions of the differential inclusion. The characterization is also extended to include functional-differential inclusions.

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**F. A. C. C. Fontes** (ffontes@math.uminho.pt), Departamento de Matemática para a Ciência e Tecnologia, Universidade do Minho, 4800-058 Guimarães, Portugal, *Continuous-Time Model Predictive Control and Discontinuous Feedback Stabilization*

Model Predictive Control (MPC) is an optimization-based control technique that has received an increasing research interest and has been widely applied in industry. Despite that, most continuous-time MPC approaches reported in literature assume continuity of the generated feedback law. Therefore, they cannot be used to stabilize important classes of nonlinear systems, such as nonholonomic systems, which frequently appear in robotics and other applications. In this talk we describe how a continuous-time MPC framework using a positive inter-sampling

time, combined with the use of an appropriate concept of solution to a differential equation, can address nonholonomic systems. The main features required for stability of such framework are reviewed. Finally, the synthesis of robust stabilizing controls under structured uncertainty is discussed.

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**Grant Galbraith** (`G.Galbraith@lse.ac.uk`), Department of Mathematics, London School of Economics, *Solution Regularity via Nonsmooth Value Functions*

We consider a calculus of variations problem with fixed endpoints and a nonsmooth, extended real-valued Lagrangian. At issue is whether we can guarantee Lipschitz regularity of an optimal solution, or to guarantee the absence of the Lavrentiev phenomenon. We will show how this can be accomplished by allowing one of the fixed endpoints to vary, then consider the behavior of the resulting value function $V$. We will see how the existence of certain types of subgradients of $V$ will help us to answer our questions of regularity in the original, fixed endpoint problem.

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**Pando G. Georgiev** (`georgiev@bsp.brain.riken.go.jp`), Laboratory for Advanced Brain Signal Processing, Brain Science Institute, The Institute of Physical and Chemical Research (RIKEN), 2-1, Hirosawa, Wako-shi, Saitama, 351-0198, JAPAN (On leave from Sofia University "St. Kl. Ohridski", Faculty of Mathematics and Informatics, Sofia, Bulgaria), *Parametric Variational Principles in Optimal Control Problems*

We present applications of parametric variational principles (Ekeland's and Borwein-Preiss' variational principles) to optimal control problems. We show that, in some classical optimal control problems, there exists a solution depending continuously on a parameter. Some of the results are based on the following lemma, which has independent interest.

*Lemma (Parametric $\varepsilon$-variational principle)* Suppose that $X$ is a paracompact topological space, $E$ is a Banach space, $Y \subset E$ is a closed, convex and nonempty subset, $F : X \to 2^Y$ is lower semi-continuous multivalued mapping with convex nonempty images, $\varepsilon > 0$ is given and the functions $f : X \times Y \to \mathbf{R}$, $g : X \to \mathbf{R}$ satisfy the conditions:

(1) $f(x, .)$ is quasi-convex for every $x \in X$;
(2) $f(., y)$ is upper semi-continuous for every $y \in Y$;
(3) $g$ is lower semi-continuous and $g(x) \geq \inf\{f(x, y) : y \in (F(x) + \varepsilon B) \cap Y\}$ for every $x \in X$, where $B$ is the closed unit ball.

Then there exists a continuous selection $\varphi_\varepsilon : X \to Y$ of the mapping $F_\varepsilon(x) = F(x) + \varepsilon B$ such that $f(x, \varphi_\varepsilon(x)) < g(x) + \varepsilon \quad \forall x \in X$.

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**Rafal Goebel** (`rafal@xanadu.ece.ucsb.edu`), Department of Electrical and Computer Engineering, University of California, Santa Barbara, *Duality and Uniqueness of Convex Solutions to Stationary Hamilton-Jacobi Equations*

One of the key issues in dynamic programming is the description of the optimal value function as the unique solution of an appropriate Hamilton-Jacobi equation. When the problem defining the value function is convex, duality theory suggests pairing it with a dual problem, possibly leading to conjugacy relationships between the original and the dual value function. In the talk, we discuss the connections between these seemingly unrelated issues, in the framework of convex control problems on infinite time intervals. Close ties between the uniqueness of convex solutions to a Hamilton-Jacobi equation, the uniqueness of convex solutions to a dual Hamilton-Jacobi equation, and the conjugacy of the primal and dual value functions will be displayed. Simultaneous approximation of the primal and dual problems by a pair of dual to each other finite horizon problems will pave the way to sufficient conditions for the mentioned uniqueness and conjugacy. Little regularity of the underlying cost functions will be required, and consequently, the Hamiltonians in question need not display any strict convexity, and may have several saddle points.

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**Lars Grüne** (`lars.gruene@uni-bayreuth.de`), University of Bayreuth, Germany, *Qualitative and Quantitative Aspects of the Input–To–State Stability Property*

In this talk we present results on the input–to–state stability (ISS) property for nonlinear perturbed systems. In the first part, we investigate the qualitative nature of this property. The main result is a theorem obtained together with E. Sontag and F. Wirth, which states that the ISS property is equivalent to the nonlinear $H_\infty$ property under suitable nonlinear changes of coordinates.

In the second part we focus on quantitative aspects of ISS. We introduce a variant of ISS, called input–to–state dynamical stability (ISDS), which utilizes a 1d dynamical system in order to describe the decay both of large initial values and of past disturbances. Here the main result is that ISDS allows for a gain preserving Lyapunov function characterization which in particular allows to give estimates for the ISDS (and ISS) robustness gain. As applications we consider a quantitative version of a nonlinear small gain theorem and results from numerical stability analysis.

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**Zhong-Ping Jiang** (`zjiang@control.poly.edu`), Department of Electrical and Computer Engineering, Polytechnic University, Brooklyn, NY 11201, *Control of Interconnected Nonlinear Systems: A Small-Gain Viewpoint*

The small-gain method for robust nonlinear control is reviewed and is further ameliorated to cover important classes of interconnected systems in dynamically perturbed, decentralized, or discrete time forms. Both partial-state and output feedback control problems are considered. It is shown that assumptions required in related but independent work of others can be significantly relaxed. An application to ship control will be shown as an illustrative example.
*Acknowledgements:* It is a pleasure to thank I. Mareels, L. Praly, A. Teel and Y. Wang for fruitful collaboration on some of the topics discussed here.

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**Matthias Kawski** (kawski@asu.edu), Department of Mathematics and Statistics, Arizona State University, Tempe, Arizona 85287, *Nonconvex Needle Variations*

We discuss a recently found counter-example (joint work with R. M. Bianchini) which shows that, in general, sets of tangent vectors generated by needle variations may fail to lack desirable convexity properties, even for the most benign systems. Our system is a four-input polynomial cascade system in a six-dimensional state space. Explicit constructions show that certain two-dimensional cross-sections of the sets of tangent vectors generated by needle variations are unions of two non-identical half-spaces [*sic*].

The focus of this presentation is on the mechanism that practically *fixes* the needle variations at a point: Recall that smooth dependence on initial conditions allows that a family of needle variations can be moved by a distance that decreases to zero, and still generate the same tangent vector. We show that the key mechanism in our example is that either of two families of needle variations – which cannot be combined – can only be moved by an amount that decreases at a rate faster than linearly to zero when compared to length of the needle variation.

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**Miroslav Krstic** (krstic@ucsd.edu), Department of Mechanical & Aero. Eng., University of California, San Diego, *Infinite Dimensional Backtepping for Boundary Control of Parabolic PDE's*

Boundary stabilization of a broad class of linear parabolic partial integro-differential equations is considered via strictly infinite dimensional backstepping, independent of any spatial discretization. The problem is formulated as a design of an integral operator whose kernel is shown to satisfy a well posed hyperbolic P(I)DE. This P(I)DE is then converted to an equivalent integral equation and, by applying a Peano-Baker-like approximation, a unique smooth solution for the kernel is found. For important special cases, feedback laws are constructed explicitly. It is also shown how to extend the approach to design inverse optimal controllers, and how to use adaptation to minimize the gain of the inverse optimal controllers (starting the gain from zero and raising it to a sufficient value to achieve stability). Finally, output-feedback boundary controllers (actuation on one boundary, sensing on the other boundary) are designed and the corresponding compensators' transfer functions (which are not rational) are found explicitly.

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**Yu. S. Ledyaev*** and **Q. J. Zhu** ({ledyaev, zhu}@wmich.edu), Department of Mathematics, Western Michigan University, Kalamazoo, MI 49008, USA, *Multidirectional Mean Value Inequalities and Weak Monotonicity*

Multidirectional mean value inequalities provide estimates of the difference of the extremal value of a function on a given bounded set and its value at a given point in terms of its (sub)-gradient at some intermediate point. We demonstrate that such multidirectional mean value inequalities and their generalizations can be obtained by using sufficient conditions for the approximate weak monotone decrease of a function along approximate trajectories of differential inclusions which allows us to remove a traditional assumption of lower boundedness on the function. We also obtain criteria for the approximate weak monotonicity and $r$-growth of lower semicontinuous functions using some discontinuous feedback constructions.

**Yuri Levin*** (ylevin@business.queensu.ca), Queen's University, Kingston, ON, K7L 3N6, CANADA, and **Adi Ben-Israel** (bisrael@rutcor.rutgers.edu), Rutgers University, Piscataway, NJ, 08854, USA, *The Newton Bracketing Method for Minimizing Convex Functions*

Let $f$ be a convex function of $n$ variables, bounded below, with infimum attained. A bracket is an interval [L,U] containing the attained infimum min $f$. The Newton Bracketing (NB) method [1] is an iterative method that at each iteration does a single Newton iteration and then reduces the bracket by either raising $L$ or lowering $U$. An initial lower bound $L$ must be provided, and the initial upper bound is $U(x)$ where $x$ is the initial iterate.

Unlike gradient methods (and other iterative methods that directly drive $x$ to optimum), the NB method has a natural stopping criterion: the bracket size. Numerical experience reported here shows the average bracket reduction per iteration is about 1/2, so convergence is fast.

The method is valid for $n = 1$. It is valid for $n > 1$ (using the directional Newton iteration [2]) provided the level sets of $f$ are not "too narrow". A precise statement for quadratic $f$, i.e., $f = \frac{1}{2}x^T Qx +$ linear terms, with $Q$ positive definite, is: the NB method is valid if the condition number of $Q$ is $< 1/(7 - \sqrt{48})$ ($\sim 14$).

The method was applied in [1] for solving location (Fermat-Weber) problems, and is applied here to location problems with affine constraints, and to affinely constrained convex programs in general. Applications to trust region subproblems in SDP are developed by H. Wolkowicz.

[1] Y. Levin and A. Ben-Israel, "The Newton Bracketing Method for Convex Minimization," *Computational Optimization and Applications*, **21**(2002), pp. 213-229.

[2] Y. Levin and A. Ben-Israel, "Directional Newton Methods in n Variables," *Mathematics of Computation*, **71**(2002), pp. 237-250.

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**Michael Malisoff*** (malisoff@lsu.edu), Department of Mathematics, Louisiana State University, Baton Rouge, LA 70803; **Ludovic Rifford** (rifford@desargues.univ − lyon1.fr), Institut Girard Desargues, Université Lyon 1, Bâtiment Braconnier, 21 Avenue Claude Bernard, 69622 Villeurbanne Cedex, France; and **Eduardo Sontag** (sontag@control.rutgers.edu), Department of Mathematics, Rutgers-New Brunswick, Hill Center-Busch Campus, 110 Frelinghuysen Road, Piscataway NJ 08854-8019, *Remarks on Input to State Stabilization*

We announce a new construction of a stabilizing feedback law for nonlinear globally asymptotically controllable (GAC) systems. Given a control affine GAC system, the feedback renders the closed loop system input to state stable with respect to actuator errors and observation errors, using sampling and Euler solutions. We also announce a variant of this result for fully nonlinear GAC systems. The proofs of our results are based on a recent construction of semiconcave Lyapunov functions.

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**William M. McEneaney** (wmceneaney@ucsd.edu), Dept. of Mathematics and Dept. of Mechanical and Aerospace Eng., University of California, San Diego, *A Max–Plus Method for Bellman Equations via Summation of Dual-Space Operators*

The solution of nonlinear optimal control problems and many nonlinear $H_\infty/L_2$-gain control problems can be obtained via solution of the corresponding Hamilton-Jacobi-Bellman (HJB) partial differential equations (PDE's). These HJB PDE's are fully nonlinear and first-order. Interestingly, the semi-groups associated with with these HJB PDE's are linear over the either the max-plus or min-plus algebra (a point which the author believes was first pointed out by Maslov in 1987). For simplicity, we discuss only the max-plus case. This led to development of a class of numerical methods which are essentially max-plus spectral methods (documented in various papers not included here due to space limitations). This class of methods is distinct from the well-known finite element and characteristic-based methods.

Here we take a different approach. To put the following discussion in perspective, note that in the linear/quadratic case, the solutions of the HJB PDE's are given simply by the solution of (finite-dimensional) Riccati equations. We will use the solutions of these simple problems to build approximate solutions of more complex ones, via max-plus summation of some associated operators over the semi-convex dual space.

In this abstract, we consider only the subclass consisting of steady-state HJB PDE's over all of $\mathbf{R}^n$. Let $S^k_\tau$ be the semi-group (for time propagation $\tau$) associated with the $k^{th}$ HJB PDE in a set of $K$ such (steady-state) PDE's. The solution of the PDE is also the fixed point of $W = S^k_\tau[W]$ (for any $\tau > 0$), or equivalently by noting the max-plus linearity of $S^k_\tau$, the max-plus eigenvector of the max-plus linear operator corresponding to max-plus eigenvalue zero.

The solutions lie in the space of semiconvex functions which we will view as a space over the max-plus algebra as opposed to the standard field. Let this space be spanned by basis functions $\{\psi_i\}$. Truncating this expansion at $M$ terms, an approximate solution is given by $W^{k,\tau} = \bigoplus_{i=1}^M e^k_i \otimes \psi_i$. Further, the vector of $e^k_i$'s satisfies the finite-dimensional eigenvector problem $\vec{e}^{\,k} = B^{k,\tau} \otimes \vec{e}^{\,k}$ where $B^{k,\tau}$ is an $M \times M$ matrix associated with $S^k_\tau$. Note that in the linear/quadratic case, $B^{k,\tau}$ is relatively easy to compute.

Define $\bar{S}_\tau[\phi] = \max_{k\in\{1,2,\ldots K\}} S^k_\tau[\phi]$. Let $\overline{B}$ be the $M \times M$ matrix associated with $\bar{S}_\tau$. Then $\overline{B} = \bigoplus_{k=1}^K B^{k,\tau}$. Thus the solution of $W = \bar{S}_\tau[W]$ is approximately given by the vector of coefficients satisfying $\overline{\vec{e}} = \overline{B} \otimes \overline{\vec{e}} = (\bigoplus_{k=1}^K B^{k,\tau}) \otimes \overline{\vec{e}}$.

Returning to the PDE's, let the $k^{th}$ PDE be $0 = H^k(x, \nabla W)$. Then consider the PDE $0 = \overline{H}(x, \nabla W) \doteq \max_{k\le K} H^k(x, \nabla W)$. Taking $\tau$ small and $M$ large, we prove that the solution $\overline{W}$ of $0 = \overline{H}(x, \nabla W)$ can be arbitrarily closely approximated on compact sets by the function $\bigoplus_{i=1}^M \overline{e}_i \otimes \psi_i$. In the case where each of the $H^k$ correspond to linear/quadratic problems, $\overline{B}$ is relatively easy to compute, and this leads to an approach for solution of HJB PDE's which are given (or approximated by) maxima of linear/quadratic HJB PDE's.

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**Boris Mordukhovich** (`boris@math.wayne.edu`), Department of Mathematics, Wayne State University, Detroit, MI, *Optimal Control of Differential-Algebraic Inclusions*

This paper deals with optimal control problems for dynamical systems governed by *delayed differential-algebraic inclusions* of the type:

$$\text{minimize} \;\; J[x,y] := \varphi(x(a), x(b)) + \int_a^b f(x(t), x(t-\Delta), \dot{y}(t))dt$$

over feasible arcs $\{x(\cdot), y(\cdot)\}$ satisfying the constraints

$$\begin{aligned}
&\dot{y}(t) \in F(x(t), x(t-\Delta), t) \;\; \text{a.e.} \;\; t \in [a,b],\\
&y(t) = x(t) + Ax(t-\Delta), \quad t \in [a,b].\\
&x(t) = c(t), \quad t \in [a-\Delta, a),\\
&(x(a), x(b) \in \Omega \subset \mathbf{R}^{2n}.
\end{aligned}$$

Such systems are important for many engineering, economic, and ecological applications. Mathematically they are essentially different from control systems governed by ordinary differential equations and inclusions as well as by those containing time delays only in state variables. In particular, an analogue of the Pontryagin maximum principle does not hold for the differential-algebraic control systems under consideration, even in the unconstrained case with smooth dynamics and no delays, without *a priori* convexity assumptions on admissible velocity sets.

Our main goal is to derive necessary optimality conditions for general optimal control problems governed by differential-algebraic inclusions with endpoint constraints. To achieve this goal, we develop the method of discrete approximations that is certainly of independent interest. It allows us to build a well-posed family of discrete approximations of the original problem and establish stability results on the strong convergence of optimal solutions. Employing powerful tools of modern variational analysis and generalized differentiation, we derive necessary optimality conditions for discrete-time systems and then, by passing to the limit as the stepsize of discretization goes to zero, we arrive at new necessary optimality conditions for the original differential-algebraic systems in both Euler-Lagrange and Hamiltonian types. This talk is based on a joint work with Lianwen Wang. This research was partly supported by the National Science Foundation under grant DMS-0072179.

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**Todd D. Murphey** (t − murphey@northwestern.edu), Mechanical Engineering, Northwestern University, 2145 Sheridan Road, Evanston, IL 60208, *Controllability and Kinematic Reductions for Overconstrained Mechanical Systems*

Controllability and reduction theory have both been extensively studied for smooth mechanical systems. However, nonsmooth mechanical systems have not received as much attention in this context. In particular, systems can potentially have more nominal constraints (both holonomic and nonholonomic) than they have degrees of freedom, thereby ensuring that at least some of these constraints cannot be satisfied. In some cases, particularly those involving friction, determining which constraints are satisfied can be very difficult and sensitive to uncertainty in the model. For instance, the Mars rover has sufficiently many nonholonomic constraints that the only motion it has which satisfies all of its constraints is the straight forward motion. Once the wheels are turned, some wheels must slip. At the same time, we can expect the Mars rover to share characteristics with the kinematic car studied frequently in the nonlinear control community. Properties such as controllability and reducibility are well established for the kinematic car, and we would like them to be useful for the rover as well. We have been developing tools for the purpose of analysis and control design for overconstrained systems (such as the Mars rover and some MEMS problems). I'll give an overview of our recent results including the use of set-valued Lie brackets for controllability analysis and a necessary and sufficient condition for kinematic reducibility for nonsmooth mechanical systems.

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**Chadi Nour** (chadi@igd.univ − lyon1.fr), Institut Girard Desargues, Université Lyon 1 (La Doua), 21 avenue Claude Bernard, 69622 Villeurbanne Cedex, France, *Nonconvex Duality in Optimal Control*

Necessary and sufficient optimality conditions are obtained for a class of optimal control problems. We find a representation of the minimum cost in terms of the upper envelope of generalized semisolutions of the Hamilton-Jacobi equation, and as a corollary a representation in terms of smooth subsolutions, generalizing a result due to Vinter. An important application of our result is a new characterization of the minimal time function.

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**Y. Orlov** (yorlov@cicese.mx), CICESE Research Center, P.O. Box 434944 San Diego, CA 92143-4944, *Control Applications of Schwartz' Distributions in Nonlinear Setting*

Schwartz' distributions theory is developed in a nonlinear setting. In order to describe complex dynamic systems with impulsive inputs, the meaning of differential equations in distributions is extended. Generalized solutions for these equations are introduced via the closure, in a certain topology, of the set of the conventional solutions corresponding to standard integrable inputs. Mathematical models proposed involve nonlinear and, generally speaking, non single-valued operating over distributions. The instantaneous impulse response of a nonlinear system is shown to depend on the impulse realization. The complete integrability of a certain auxiliary system appears to guarantee the uniqueness of the impulse response. The theory is demonstrated to be eminently suited to optimal impulsive feedback synthesis, filtering of stochastic and deterministic dynamic systems over sampled-data measurements, and the analysis of mechanical systems with impulsive phenomena.

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**Norma Ortiz** (ortiz@math.lsu.edu), Department of Mathematics, Louisiana State University, Baton Rouge, LA 70803-4918, *Existence of Solutions and a Decoupling Method for a Neutral Problem of Bolza*

In this talk we present results that argue the existence of solutions to the problem of Bolza with delays in both the state and velocity variables. We use a decoupling technique, developed by Clarke for the case without delays, and introduce briefly how it can be used to develop necessary conditions for our problem. This work has been done under the supervision of the presenter's advisor, Peter Wolenski.

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**Daniel N. Ostrov** (dostrov@scu.edu), Department of Mathematics and Computer Science, Santa Clara University, 500 El Camino Real, Santa Clara, CA 95053-0290, *Nonuniqueness in Systems of Hamilton-Jacobi Equations*

The scalar equation $u_t + H(f(x,t), u_x) = 0$ where $H$ is convex and grows superlinearly in $u_x$ has the well known property that optimal control paths for its solution (i.e., characteristic paths) cannot enter a shock as the path progresses towards the equation's initial condition if $f$ is a continuous function. On the other hand these paths can enter shocks if $f$ is a discontinuous function. This phenomenon will allow us to see how the solution for some viscous non-strictly hyperbolic decoupled systems of the form

$$\begin{aligned} u_t + H_1(u_x) &= \varepsilon_1 u_{xx} \\ v_t + H_2(u_x, v_x) &= \varepsilon_2 v_{xx}, \end{aligned}$$

is not unique as $(\varepsilon_1, \varepsilon_2) \to (0,0)$; that is, the solution depends on how $(\varepsilon_1, \varepsilon_2)$ approach $(0,0)$. If time permits we will also examine some of the difficulties in trying to use control theory to describe unique solutions to strictly hyperbolic systems of Hamilton-Jacobi equations.

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**Daniel Pasca** (dpasca@wpi.edu), Mathematical Sciences Department, Worcester Polytechnic Institute, 100 Institute Road, Worcester, MA 01609, USA, *Periodic Solutions of Second Order Differential Inclusions*

Using an abstract framework due to Clarke we prove the existence of periodic solutions for second-order differential inclusions systems. For this we use the variational methods (direct method, nonsmooth variant of minimax theorems).

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**A. Arutyunov** (arutun@orc.ru), Peoples Friendship University of Russia, Moscow 117198, Mikluka-Maklai, 6, Russia, and **F. Lobo Pereira*** (flp@fe.up.pt), Faculdade de Engenharia da Universidade do Porto, R. Dr. Roberto Frias, 4200-465 Porto, Portugal, *Second order necessary conditions for optimal impulsive control problems*

In this presentation, we address first and second order necessary conditions of optimality for an impulsive control problem. This is not the most general formulation, but it clearly illustrates one of the main features of these results: no *a priori* normality assumptions are required, and the assumptions are informative for abnormal control processes as well.

The method used in the proof of these conditions differs substantially from all others so far adopted in the impulsive control literature. The proof consists of regarding the optimal control problem as an instance of a general abstract optimization problem with equality and inequality type constraints and constraints given by convex cone, for which an extremal principle is proved for an abstract minimization problem. An outline of the proof will be sketched, and simple examples for both normal and abnormal problems will be given.

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**Franco Rampazzo*** and **Monica Motta** ({rampazzo,motta}@math.unipd.it), Dipartimento di Matematica Pura ed Applicata, Università di Padova, ITALY, *Multitime Hamilton-Jacobi systems*

We investigate existence and uniqueness of the (viscosity) solution for systems of convex

Hamilton-Jacobi equations of the form

$$\begin{cases} u_{t_1}(t_1,...,t_d,x) + H_1(x, Du(t_1,...,t_d,x)) = 0 \\ .... \\ .... \\ u_{t_d}(t_1,...,t_d,x) + H_d(x, Du(t_1,...,t_d,x)) = 0 \end{cases} \qquad (t_1,...,t_d,x) \in ]0,T[^d \times \mathbf{R}^n \tag{4}$$

satisfying the initial condition

$$u(0,...,0,x) = \psi(x)$$

These systems are called *multi-time* in an earlier paper by P.Lions and J.C.Rochet. An application in economics of these system is presented in [R].

In general system (4) is overdetermined. Actually this is not the case when the Hamiltonians $H_i$ do not depend on the state variable $x$. In fact, this case is investigated in [LR], where the authors utilize a commutativity property for semigroups to prove existence of a *weak* solution to (4). Their tools are essentially Oleinik-Lax formulas. A generalization of this result to the case when the Hamiltonians depend un $u$ as well (but not on $x$) has been recently provided by S. Plaskacz and M. Quincampoix in [PQ]. They exploit Oleinik-Lax formulas up to the point they can allow semicontinuous solutions.

Of course, the problem with $x$-dependent Hamiltonians has a richer underlying geometry, and, to our knowledge, it has been investigated only in a recent paper by G. Barles and A. Tourin [BT]. They prove existence of a viscosity solution under the assumptions that the Hamiltonians are $C^1$ and convex in the gradient variable, and, moreover, the involution property

$$\{H_i, H_j\} = (H_i)_x(H_j)_p - (H_j)_x(H_i)_p = 0 \qquad \forall i,j = 1,...d \tag{5}$$

is verified on the whole domain.

Of course hypothesis (5) is satisfied in the $x$-independent case. Furthermore it can be shown that (5) is necessary for the existence of even a local solution of (4). However, as the same authors point out, with the methods exploited in [BT] the $C^1$-regularity assumption is hardly removable.

Here we are able to handle a class of *Lipschitz continuous* Hamiltonians arising in Control Theory (moreover, we can drop a coercivity- type condition assumed in [BT] mostly for technical reasons). Our approach is based on a combination of control-theoretical techniques and arguments from the theory of viscosity solutions. Moreover, in order to allow nonsmoothness, we utilize a recent result by F. Rampazzo and H. Sussmann on the commutativity of Lipschitz continuous vector fields.


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**Alain Rapaport*** (rapaport@ensam.inra.fr), UMR Analyse des Systèmes et Biométrie, Institut National de la Recherche Agronomique (INRA), 2 place Viala, 34060 Montpellier, France, and **Pierre Cartigny** (cartigny@ehess.cnrs − mrs.fr), Le Groupement de Recherche en Économie Quantitative d'Aix-Marseille (GREQAM), Université de la Méditerranée, 2 rue de la Vieille Charité, 13002 Marseille, France, *Turnpike Theorems in Infinite Horizon by a Value Function Approach*

We consider a problem of calculus of variations in infinite horizon whose objective $J(.)$ is given by

$$J(x(.)) = \lim_{T\to+\infty} \int_0^T e^{-\delta t} l(x(t), \dot{x}(t)) dt$$

where $\delta$ is a positive number and $l$ is a real valued function on $\mathbb{R} \times \mathbb{R}$, linear with respect to its second argument. Our interest is the maximization of $J$ on the paths $x(.)$ with fixed initial condition $x(0) = x_0$, for which the preceding improper integral converges.

If the theory of calculus of variations is well established when the horizon is finite, the situation is not the same when one deals with infinite horizon. For instance, Ekeland (Lecture Notes, 1986) listed a collection of open problems in this setting, and underlined that the standard optimality conditions, such as the Euler first-order condition, have not yet been established in this framework. Our goal in this work is to study the so called Turnpike Property, which asserts, roughly speaking, that there exists a particular solution $\bar{x}(.)$ (called the "turnpike") such that, from any initial condition, an optimal trajectory reaches the path $\bar{x}(.)$ as quickly as possible.

We focus on the singular scalar case (i.e., the case where the Euler equation degenerates in an algebraic system), assuming that the integrand $l(x, y)$ is linear with respect to $y$ and that $y \in [\alpha, \beta]$. We propose a new optimality condition of the MRAPs (Most Rapid Approach Path) which is necessary and sufficient. We consider more general growth assumptions than the usual ones. Our approach is based on a characterization of the value function of a particular Hamilton-Jacobi equation, in terms of viscosity solutions. This approach also allows us to consider the case of a multiplicity of singular solutions of the Euler equation, and therefore provides a criterion for the choice of the turnpikes in competition, depending on the initial condition. Finally we give an example which exhibits the different possible occurrences of turnpikes (one or several).

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**Ludovic Rifford** (rifford@igd.univ − lyon1.fr), Institut Girard Desargues, Université Lyon I, 21 avenue Claude Bernard, 69622 Villeurbanne Cedex, France, *The Stabilization Problem: AGAS and SRS Feedbacks*

Given an asymptotically controllable control system, we present existence results of two kind of interesting stabilizing feedbacks. First, a feeedback is said to be Almost Globally Asymptotically Stabilizing (AGAS) if the associated closed-loop system stabilizes almost every point to the equilibrium (with Lyapunov stability). Using the notion of stratified semiconcave control-Lyapunov functions, we will explain how to prove that every asymptotically controllable system admits an AGAS feedback. Second, we will introduce the concept of smooth repulsive stabilizing (SRS) feedbacks, and give first existence results in the framework of locally controllable control systems (that is control systems wich satisfy the Chow's condition) on smooth manifolds.

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**Vinicio R. Rios** (rios@math.lsu.edu), Mathematics Department, Louisiana State University, Baton Rouge, Louisiana 70803, USA, *A Characterization of Strongly Invariant Systems for a Class of non-Lipschitz Multifunctions*

The goal of this talk is to present a characterization of strongly invariant systems where the multifunction defining the system is the sum of a maximal dissipative and a Locally Lipschitz set-valued map. These results have been obtained jointly with Peter R. Wolenski who is the dissertation advisor of the presenter.

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**Ilya Shvartsman** (ilya@math.wayne.edu), Department of Mathematics, Wayne State University, Michigan, *Minimax Design of Constrained Parabolic Systems*

We consider a minimax control problem for linear parabolic systems with uncertain disturbances and pointwise constraints on state and control variables, whose dynamics is typical in applied problems. A natural approach to control design of such uncertain systems, related to H-infinity control and differential games, is minimax synthesis that guarantees the best system performance under the worst perturbations and ensures an acceptable behavior for any admissible perturbations. The design procedure essentially employs monotonicity properties of the parabolic dynamics and its asymptotics on the infinite horizon. We find a convenient suboptimal structure of feedback boundary controllers that ensures the required system performance and robust stability under any admissible perturbations. This is joint work with Boris S. Mordukhovich.

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**Ronald Stern** (stern@vax2.concordia.ca), Department of Mathematics, Concordia University, Canada, *Brockett's Necessary Condition for Stabilization: The State Constrained Case*

A variant of Brockett's necessary condition for regular feedback stabilization is provided, in the state constrained case. The proof utilizes proximal techniques, and "proximal aiming" in particular.

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**S. S. Sritharan** (sri@uwyo.edu), Department of Mathematics, University of Wyoming, Laramie, WY 82071, and **Andrzej Święch*** (swiech@math.gatech.edu), School of Mathematics, Georgia Institute of Technology, Atlanta, GA 30332, *Bellman Equations for Optimal Control of Stochastic Navier-Stokes Equations*

We will discuss results on Hamilton-Jacobi-Bellman equations associated with the stochastic optimal control of viscous hydrodynamics. These are infinite dimensional, second-order equations that involve unbounded nonlinear operators and they may be very degenerate. We will present a viscosity solution approach to such equations that guarantees existence and uniqueness of generalized solutions.

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**Vladimir M. Veliov** (`vveliov@eos.tuwien.ac.at`), Institute for Econometrics, Operations Research and Systems Theory, Vienna University of Technology, Vienna, Austria, and Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Sofia, Bulgaria, *Optimal Control of Uncertain Systems with Incomplete Information for the Disturbances*

We investigate the problem of optimization of a terminal cost function for a system depending on a control, and on two disturbances for which a *priori* set-membership is known. The disturbances are of different nature: one becomes known to the controller at the current time (we called it observable), while the other remains unknown. The problem can be viewed as a differential game of min-max type where the controller aims at minimization of the objective function by a strategy which depends only on the observable disturbance. Since the state of the system is not exactly known due to the presence of a unobservable disturbance, we reformulate the problem through a set-valued dynamics describing the evolution of the current set-estimation of the state. To reduce the complexity of the problem we pass to a sub-optimal problem where the evolution of the state estimation is restricted to a prescribed collection of sets. The main result of the paper is a characterization of the value function of this problem trough a Hamilton-Jacobi inequality in terms of Dini derivatives, which implies a convergent scheme for numerical computations. As necessary auxiliary tools we provide new results on evolution and viability of tubes in a given collection of sets, that may be of independent interest.

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**Bingwu Wang** (`bwang@emunix.emich.edu`), Department of Mathematics, Eastern Michigan University, Ypsilanti, MI, *Generalized Differentiation for Moving Objects*

This work is devoted to the analysis of moving sets and mappings commonly arising in control and optimization theory. We summarize, unify, and generalize some of the known concepts in this area, such as extended normal cone, extended coderivative and extended subdifferential, which are natural generalizations of the corresponding normal cone, coderivative and subdifferential for the case of non-moving objects, respectively. In addition, we propose a new notion of sequential normal compactness for the moving objects. We then establish a complete calculus for our constructs, under the assumption of sequential normal compactness. We also develop a full set of calculus rules for the extended normal compactness that is crucial for applications to optimization and control theory. We illustrate how most results for the non-moving situations can be generalized to the moving case. Our main tool is a fuzzy intersection rule based on the extremal principle. The work is inspired by recent results in control and optimization by Boris Mordukhovich, Jay Treiman, Qiji Zhu, and others.

This talk is based on joint work with Boris Mordukhovich.

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**Lianwen Wang** (`lwang@math.wayne.edu`), Department of Mathematics, Wayne State University, USA, *Optimal Control of Constrained Delay-Differential Inclusions with Set-Valued Tail Conditions*

The paper studies a general optimal control problem for nonconvex delay-differential inclusions with endpoint constraints. We incorporate set-valued 'tail' conditions for the initial interval, which provide an additional source for optimization. Our variational analysis is based on well-posed discrete approximations of constrained delay-differential inclusions by a family of time-delayed systems with discrete dynamics and perturbed constraints. Using convergence results for discrete approximations and advanced tools of nonsmooth variational analysis, we derive necessary optimality conditions for constrained delay-differential inclusions in both Euler-Lagrange and Hamiltonian forms, involving nonconvex generalized differential constructions for nonsmooth functions, sets, and set-valued mappings. This is joint work with Boris Mordukhovich.

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**Stanislav Žabić** (zabic@math.lsu.edu), Department of Mathematics, Louisiana State University, 327 Lockett Hall, Baton Rouge, Louisiana 70803-4918, USA, *Sampling Methods for Impulse Systems*

This talk will introduce a sampling technique for impulsive systems analogous to the Euler's method. The limits of the sampled approximations are shown to converge to the generalized solutions that was introduced by A. Bressan and F. Rampazzo. Moreover, we will see that the sampling technique generates the same type of solution as the one generated by systems with approximate controls. Further applications of the sampling technique are proposed at the end of the talk. All the results are a part of the presenter's thesis work done under the supervision of Peter R. Wolenski.

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**Zsolt Páles** (pales@math.klte.hu), University of Debrecen, Debrecen, Hungary and **Vera Zeidan*** (zeidan@math.msu.edu), Michgan State University, East Lansing, MI, *Critical Cones and Critical Tangent Cones in Optimization Problems*

The notion of second-order admissible variation defined by Dubovitskii and Milyutin in 1965 turned out to be an essential notion in the theory of second-order necessary conditions for optimum problems. It is introduced in the following definition.

Definition. Let $X$ be a normed space, $\mathbf{Q} \subset X$, $x \in \mathbf{Q}$, and $d \in X$. A vector $v \in X$ is called a *second-order admissible variation of* $\mathbf{Q}$ *at* $x$ *in the direction* $d$ if there exists $\overline{\varepsilon} > 0$ such that

$$x + \varepsilon d + \varepsilon^2(v + u) \in \mathbf{Q} \qquad \text{for all} \quad 0 < \varepsilon < \overline{\varepsilon},\ \|u\| < \overline{\varepsilon},\ u \in X.$$

The set of all such variations is denoted by $V(x, d|\mathbf{Q})$. It follows directly from the definition that $V(x, d|\mathbf{Q})$ is an open set. If $\mathbf{Q}$ is also convex, then $V(x, d|\mathbf{Q})$ is convex as well. In Levitin, Milyutin, and Osmolovskii (1978), a general discussion on second- and higher-order necessary conditions can be found.

In order to derive meaningful second-order optimality conditions for optimum problems, it is required to select directions $d$ for which the set $V(x, d|\mathbf{Q})$ is nonempty. Such directions $d \in X$ are labeled as the *critical directions of* $\mathbf{Q}$ *at* $x$ and form a set called *critical directions cone to* $\mathbf{Q}$ *at* $x$. Throughout this talk, this cone will be denoted by $C(x|\mathbf{Q})$. It can be easily seen that $C(x|\mathbf{Q})$ is a *convex cone* if $\mathbf{Q}$ is convex.

Define

$$K(x|\mathbf{Q}) := \operatorname{cone}(\mathbf{Q} - x) := \{\lambda(q - x) \mid q \in \mathbf{Q},\ \lambda > 0\},$$

and its closure

$$T(x|\mathbf{Q}) := \overline{\operatorname{cone}}(\mathbf{Q} - x) = \operatorname{cl} K(x|\mathbf{Q}).$$

If $\mathbf{Q}$ is convex, then for the nonemptiness of $V(x, d|\mathbf{Q})$ it is necessary, but not sufficient that the interior of $\mathbf{Q}$ be nonempty and $d$ belong to $T(x|\mathbf{Q})$. However, the nonemptiness of $V(x, d|\mathbf{Q})$ is assured if $\operatorname{intr} \mathbf{Q} \neq \emptyset$ and $d \in K(x|\mathbf{Q})$. Therefore, for convex $\mathbf{Q}$ with nonempty interior, we have

$$K(x|\mathbf{Q}) \subset C(x|\mathbf{Q}) \subset T(x|\mathbf{Q}).$$

In order to demonstrate the use of second-order it admissible variations, consider the following optimization problem:

$$(\mathcal{P}) \qquad \text{Minimize} \quad F(z) \quad \text{subject to} \quad G(z) \in \mathbf{Q}, \; H(z) = 0,$$

where $F : \mathcal{D} \to \mathbf{R}$, $G : \mathcal{D} \to X$, $H : \mathcal{D} \to Y$, and $X$, $Y$, $Z$ are Banach spaces, $\mathcal{D} \subset Z$ is nonempty and open, and $\mathbf{Q} \subset X$ is a closed convex set with nonempty interior.

The prototype of such problems arise, for instance, in optimal control theory with control and/or state constraints in the inclusion form $x(t) \in \mathcal{Q}(t)$. This pointwise condition would then lead to the set $\mathbf{Q}$ which consists of appropriate selections $x$. Let us recall the first- and second-order necessary conditions for $(\mathcal{P})$, obtained by the authors in 1994. For, we introduce the following notations and notions.

- A point $\hat{z} \in \mathcal{D}$ is called an *admissible point* for $(\mathcal{P})$ if $G(\hat{z}) \in \mathbf{Q}$ and $H(\hat{z}) = 0$ hold. A point $\hat{z} \in \mathcal{D}$ is a *solution (local minimum)* of the problem if it is admissible and there exists a neighborhood $U$ of $\hat{z}$ such that $F(z) \geq F(\hat{z})$ for all admissible points $z \in U$.
- A point $\hat{z} \in \mathcal{D}$ is called a *regular point* for $(calP)$ if $F$, $G$, and $H$ are strictly Fréchet differentiable at $\hat{z}$ and the range of the linear operator $H'(\hat{z})$ is a closed subspace of $Y$.

Let $\hat{z}$ be an admissible regular point for $(\mathcal{P})$ and $d \in Z$.

- A vector $\delta z \in Z$ is called a *critical direction* at $\hat{z}$ for $(\mathcal{P})$ provided that $F'(\hat{z})\delta z \leq 0$, $G'(\hat{z})\delta z \in C(G(\hat{z})|\mathbf{Q})$, and $E'(\hat{z})\delta z = 0$.
- A vector $\delta z \in Z$ is called a *regular direction* at $\hat{z}$ for $(\mathcal{P})$ if the second-order directional derivative of $L := (F, G, H)$

  $$L''(\hat{z}, \delta z) := \lim_{\varepsilon \to 0+} 2\frac{L(\hat{z} + \varepsilon\delta z) - L(\hat{z}) - \varepsilon L'(\hat{z})\delta z}{\varepsilon^2}$$

  exists.

Clearly, the *zero* vector is always a regular critical direction at $\hat{z}$ for $(\mathcal{P})$. Now we are ready to state a particular case of the result developed in 1994.

**Theorem 0.0.1** *(P/Z) Let $\hat{z}$ be a regular local solution of the above problem $(\mathcal{P})$. Then, for all regular critical directions $\delta z$, there correspond Lagrange multipliers $\lambda \geq 0$, $\xi \in X^*$, and $\eta \in Y^*$ (which depend on $\delta z$), such that at least one of them is different from zero and the following relations hold*

$$\xi \in N(G(\hat{z})|\mathbf{Q}), \tag{6}$$

$$\lambda F'(\hat{z})z + < \xi, G'(\hat{z})z > + < \eta, E'(\hat{z})z > = 0 \qquad \textit{for} \quad z \in Z, \tag{7}$$

*and*

$$\lambda F''(\hat{z}, \delta z) + < \xi, G''(\hat{z}, \delta z) > + < \eta, E''(\hat{z}, \delta z) > \quad \geq 2\delta^*(\xi|V(G(\hat{z}), G'(\hat{z})\delta z|\mathbf{Q})). \tag{8}$$

(Here $\delta^*$ stands for the support function and $N(x|\mathbf{Q})$ denotes the adjoint cone of $T(x|\mathbf{Q})$, that is the cone of outward normals to the set $\mathbf{Q}$ at the point $x$.)

The result presented in the above theorem was derived under the Mangasarian-Fromovitz condition, by Kawasaki in 1988 for the case when $\mathbf{Q}$ is a cone, and by Cominetti 1990 for the case when $\mathbf{Q}$ is convex. The possibility of removing the Mangasarian-Fromovitz condition, as is obtained in the above theorem, was noted by Ioffe 1989.

If $d \in K(x|\mathbf{Q})$, then $V(x,d|\mathbf{Q})$ is nonempty and $V(x,d|\mathbf{Q}) = \mathrm{cone}(\mathrm{cone}(\mathrm{intr}\,\mathbf{Q} - x) - d)$, that is, $V$ is a cone. Thus, if this is the case for $d := G'(\hat{z})\delta z$ and $x := G(\hat{z})$, then the right-hand side in the second-order condition inequality (3) vanishes. This is the case of the result obtained for instance by Ben-Tal and Zowe in 1982. However examples are provided by Kawasaki in 1988 that show that the necessary conditions with *nonzero extra term*, which occurs only if $d \in T(x|\mathbf{Q}) \setminus K(x|\mathbf{Q})$, handle situations that cannot be handled with previous results, that is, when $d$ is taken from $K(x|\mathbf{Q})$. Thus, one has to also consider directions $d \in T(x|\mathbf{Q}) \setminus K(x|\mathbf{Q})$. In this important case two questions naturally arise from the theorem:

(i) How can we check the nonemptiness of $V(x,d|\mathbf{Q})$, that is, how can the critical cone $C(x|\mathbf{Q})$ be characterized in terms of $\mathbf{Q}$.

(ii) How to evaluate the support function of $V(x,d|\mathbf{Q})$ in terms of that of $\mathbf{Q}$ itself.

A significant setting is the case when $\mathbf{Q}$ is a subset of the space of continuous functions, $\mathcal{C}(T,\mathbf{R}^\kappa)$, and is defined by

$$\mathbf{Q} = \mathrm{sel}_C(\mathcal{Q}) := \{x \in \mathcal{C}(T,\mathbf{R}^\kappa) \,|\, x(t) \in \mathcal{Q}(t) \text{ for all } t \in T\}, \tag{9}$$

where $\mathcal{Q}$ is a lower semicontinuous set-valued map whose images are closed, convex sets with nonempty interior, and $T$ is a compact Hausdorff space. This type of constraints represents the state constraints in control problems.

Another case of interest is when $\mathbf{Q}$ is a subset of the space of essentially bounded functions, $\mathcal{L}^\infty(\Omega,\mathbf{R}^\gamma)$, and is defined by

$$\mathbf{Q} = \mathrm{sel}_\infty(\mathcal{Q}) := \{x \in \mathcal{L}^\infty(\Omega,\mathbf{R}^\gamma) \,|\, x(t) \in \mathcal{Q}(t) \text{ for a.e. } t \in \Omega\}, \tag{10}$$

where $\mathcal{Q}$ is a measurable set-valued map whose images are closed and have nonempty interior, and $(\Omega,\mathcal{A},\nu)$ is a complete finite measure space. This type of constraints is typical for control constraints in control problems.

However, there are optimization problems for which the constraint set $\mathbf{Q}$ in neither a subset of $\mathcal{C}(T,\mathbf{R}^\kappa)$ nor a subset of $\mathcal{L}^\infty(\Omega,\mathbf{R}^\gamma)$, but a subset of a general normed space $X$.

The main goal of this talk is to answer the above questions (i)-(ii) when the underlying space $X$ is *any* normed space. For the special case when $\mathbf{Q}$ is given by (4) or (5), we wish the answer to (i)-(ii) be phrased in terms of the original data, that is, the set-valued map $\mathcal{Q}$.

To reach the first goal, a characterization of the critical cone $C(x|\mathbf{Q})$ is derived in terms of the set $\mathbf{Q}$. This leads to the introduction of $CT(x|\mathbf{Q})$, the *critical tangent cone* of a set $\mathbf{Q} \subset X$ at $x$. Properties and examples of this cone will be presented. For the second goal, we managed to develop in terms of the support function of $\mathbf{Q}$, a *formula* for the extra term in inequality (3), that is, for the support function $V(x,d|\mathbf{Q})$ at $\xi \in X^*$. When $\mathbf{Q}$ is given by either (4) or (5), the characterization of $C(x|\mathbf{Q})$ simplifies drastically. Furthermore, in these cases, the extra term can then be expressed in terms of the support function of the images of the set-valued map $\mathcal{Q}$ as long as in the case of equation (5) the extra term in (3) is to be evaluated at $\xi \in \mathcal{L}^1(\Omega,\mathbf{R}^\gamma)$.

The proofs of these results are stimulated by a string of papers that the authors wrote in the last decade on the special cases considered in equations (4) and (5). However, in those cases they were able to reduce the infinite dimension feature of questions (i)-(ii) to one of finite dimension through the images $\mathcal{Q}(t)$ of the set-valued map $\mathcal{Q}$. In the general setting, which is the issue in this talk, no such structure is given for $\mathbf{Q}$, but nevertheless answers to (i)-(ii) can still be provided.

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**Yuri S. Ledyaev** and **Qiji J. Zhu*** ({`ledyaev`, `ZHU`}@`wmich.edu`), Department of Mathematics, Western Michigan University, Kalamazoo, MI, *Nonsmooth Analysis on Smooth Manifolds*

We study infinitesimal properties of nonsmooth (nondifferentiable) functions on smooth manifolds. The eigenvalue function of a matrix on the manifold of symmetric matrices provides a natural example of such nonsmooth function. In this talk we discuss how to use variational methods to derive subdifferential calculus for lower semicontinuous functions on smooth manifolds and illustrate its applications with selected examples including the calculation of subdifferentials for eigenvalue functions, constrained optimization problems on manifolds, generalized solutions of first-order partial differential equations on manifolds, and a criterion for monotonicity and invariance of functions and sets with respect to solutions of differential inclusions.

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